"FRAPPY"

{Free Response AP Problem...Yay!}

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 25 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Two treatments, A and B, showed promise for treating a potentially fatal disease. A randomized experiment was conducted to determine whether there is a significant difference in the survival rate between patients who receive treatment A and those who receive treatment B. Of 154 patients who received treatment A, 38 survived for at least 15 years, whereas 16 of the 164 patients who received treatment B survived at least 15 years.

Scoring:

(a) Treatment A can be administered only as a pill, and treatment B can be administered only as an injection. Can this randomized experiment be performed as a double-blind experiment? Why or why not?

E P I

(b) The conditions for inference have been met. Construct and interpret a 95 percent confidence interval for the difference between the proportion of the population who would survive at least 15 years if given treatment A and the proportion of the population who would survive at least 15 years if given treatment B.

E P I

In many of these types of studies, physicians are interested in the ratio of survival probabilities, $\frac{p_A}{p_B}$, where p_A represents the true 15-year survival rate for all patients who receive treatment A and p_B represents the true 15-year survival rate for all patients who receive treatment B. This ratio is usually referred to as the relative risk of the two treatments.

For example, a relative risk of 1 indicates the survival rates for patients receiving the two treatments are equal, whereas a relative risk of 1.5 indicates that the survival rate for patients receiving treatment A is 50 percent higher than the survival rate for patients receiving treatment B. An estimator of the relative risk is the ratio of estimated probabilities, $\frac{\hat{p}_A}{\hat{p}_B}$.

(c) Using the data from the randomized experiment described above, compute the estimate of the relative risk.

The sampling distribution of $\frac{\hat{p}_A}{\hat{p}_B}$ is skewed. However, when both sample sizes n_A and n_B are relatively large, the distribution of $\ln\left(\frac{\hat{p}_A}{\hat{p}_B}\right)$ - the natural logarithm of relative risk - is approximately normal with a mean of $\ln\left(\frac{p_A}{p_B}\right)$ and a standard deviation of $\sqrt{\frac{1-p_A}{n_A p_A}+\frac{1-p_B}{n_B p_B}}$, where p_A and p_B can be estimated by using $\hat{p}_A$ and $\hat{p}_B$.

When a 95 percent confidence interval for $\ln\left(\frac{p_A}{p_B}\right)$ is known, an approximate 95 percent confidence interval for $\frac{p_A}{p_B}$ - the relative risk of the two treatments - can be constructed by applying the inverse of the natural logarithm to the endpoints of the confidence interval for $\ln\left(\frac{p_A}{p_B}\right)$.

(d) The conditions for inference are met for the data in the experiment above, and a 95 percent confidence interval for $\ln\left(\frac{p_A}{p_B}\right)$ is (0.3868, 1.4690). Construct and interpret a 95 percent confidence interval for the relative risk, $\frac{p_A}{p_B}$, of the two treatments.

E P I

(e) What is an advantage of using the interval in part (d) over using the interval in part (b)?

E P I

Total:__/4